METHOD OF CALCULATING THE DIELECTRIC LOSS TANGENT OF BINARY SYSTEMS

G. N. Dul'nev, D. P. Volkov,
and M. V. Dobrochasov

UDC 621.317.33:538.569

Using a model structure with interpenetrating components, we derive formulas for the dielectric loss tangent.

It was noted in [1] that there are many different physical processes described by an equation of the type

$$\mathbf{A} = \Lambda \mathbf{B}, \tag{1}$$

where **A** and **B** are vector quantities, and Λ is a transport coefficient or other physical property of the material. Such well-known relations as the laws of Ohm, Hooke, and Fick can be written in the form (1).

The dielectric constant ε is also defined by an equation of the type (1) [2]:

$$\mathbf{D} = -\varepsilon \mathbf{E}, \tag{2}$$

relating the electric induction vector **D** to the electric field vector. The dielectric loss tangent tan δ can be expressed in terms of the dielectric constant ε and the electric conductivity σ by the equation [3]

$$\operatorname{tg}\delta = \frac{\sigma}{\omega\varepsilon} = \frac{1}{\omega RC}. \tag{3}$$

Therefore the determination of σ, ε, and tan δ can be reduced to the calculation of the generalized conductivity coefficient Λ and thus the methods of the theory of generalized conductivity can be used [1]. We consider the class of materials whose structure can be represented in the form of interpenetrating components. We consider only mechanical mixtures and porous materials in which the skeleton and pores form a structure with interpenetrating components. An elementary cell of such a structure is shown in Fig. 1a. The inputs to the calculation are the electrical conductivity, the dielectric constants of the components, and their volume concentration (porosity). In [1] analytical expressions for the conductivity $\Lambda = \sigma = 1/\rho$ and the dielectric constant $\Lambda = \varepsilon$ were obtained:

$$\Lambda = \Lambda_1 \left[x^2 + \nu (1-x)^2 + \frac{2x\nu (1-x)}{\nu x + 1 - x} \right], \quad \nu = \frac{\sigma_2}{\sigma_1}, \quad \frac{\varepsilon_2}{\varepsilon_1}. \tag{4}$$

Here x is the dimensionless parameter of the model $x = \Delta/L$, which is related to the volume concentration (porosity) m_2 of the second component by the equation [1]

$$2x^3 - 2x^2 + 1 = m_2, \quad m_1 + m_2 = 1. \tag{5}$$

We substitute in (3) the values of the coefficients ε and σ from (4) and obtain the dielectric loss tangent of the material. However data for both the electrical resistivity and dielectric constant do not exist for all materials. This creates a difficulty in the calculation. Expression (3) can be rewritten in a different form:

$$C = \frac{1}{\omega \operatorname{tg}\delta R} = \frac{S}{\omega L} \frac{1}{\rho \operatorname{tg}\delta}, \tag{6}$$

$$R = \frac{1}{\omega \operatorname{tg}\delta C} = \frac{L}{\omega S} \frac{1}{\varepsilon \operatorname{tg}\delta}. \tag{7}$$

Leningrad Institute of Exact Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 3, pp. 425-429, March, 1987. Original article submitted July 10, 1985.

 0022-0841/87/5203-0310$12.50

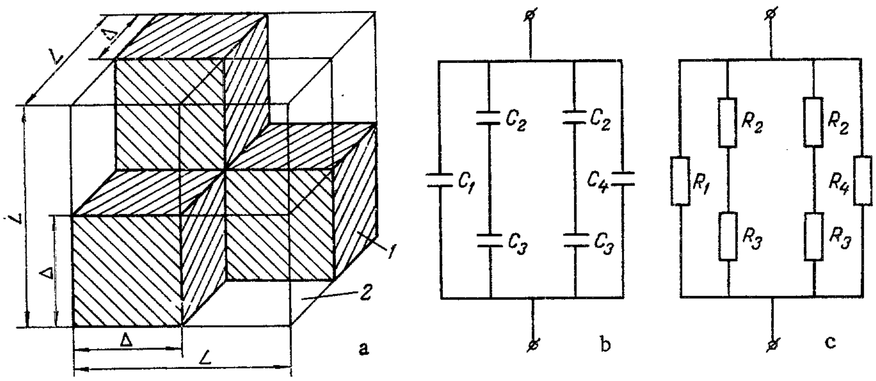

Fig. 1. (a) Elementary cell of a structure with interpenetrating components: 1) tan δ_1; 2) tan δ_2; (b) combination of capacitors; (c) and resistances.

In analogy with [1], we consider the capacitance of the elementary cell shown in Fig. 1a. We place it between the plates of a capacitor in a stationary electric field and find the capacitance of the resulting compound capacitor. We divide the elementary cell into parts using a system of infinitely thin plates which are impermeable to current. This operation leads to the linearization of the potential in each part and its capacitance C_i becomes proportional to its cross-sectional area S_i and inversely proportional to the length L of the cell in the direction of the field and to the product of the electrical resistivity and the dielectric loss tangent ρ_i tan δ_i (equation (6)). The total capacitance of the composite capacitor is (Fig. 1b)

$$C = C_1 + C_4 + 2\frac{C_2C_3}{C_2+C_3}. \tag{8}$$

On the other hand, the total capacitance of the elementary cell is, by definition,

$$C = \frac{L^2}{\omega L}\frac{1}{\rho \operatorname{tg}\delta}. \tag{9}$$

Equating (8) and (9) and substituting the expressions for the capacitances C_i of the separate i-th parts:

$$C_1 = \frac{\Delta^2}{\omega L}\frac{1}{\rho_1 \operatorname{tg}\delta_i},\quad C_3 = \frac{(L-\Delta)}{\omega}\frac{1}{\rho_2 \operatorname{tg}\delta_2},$$
$$C_2 = \frac{\Delta}{\omega}\frac{1}{\rho_1 \operatorname{tg}\delta_1},\quad C_4 = \frac{(L-\Delta)^2}{\omega L}\frac{1}{\rho_2 \operatorname{tg}\delta_2} \tag{10}$$

we obtain the following expression for the dielectric loss tangent:

$$\frac{1}{\operatorname{tg}\delta} = \frac{\rho}{\rho_1 \operatorname{tg}\delta_1}\left[x^2 + \nu_1(1-x)^2 + \frac{2x\nu_1(1-x)}{\nu_1 x + 1 - x}\right],\quad \nu_1 = \frac{\rho_1 \operatorname{tg}\delta_1}{\rho_2 \operatorname{tg}\delta_2}. \tag{11}$$

We now represent an elementary cell as a combination of resistors (Fig. 1c). The total resistance of the elementary cell is equal to the sum of the resistances of the separate parts:

$$R^{-1} = R_1^{-1} + R_4^{-1} + 2/(R_2+R_3). \tag{12}$$

The total resistance of the elementary cell is given by (7) assuming that the entire volume is filled with an isotropic medium with an effective dielectric constant ε and an effective dielectric loss tangent tan δ. Equating (7) and (12) and using the resistances R_i of the separate i-th parts, we obtain the following expression for the dielectric loss tangent:

$$\operatorname{tg}\delta = \frac{\operatorname{tg}\delta_1\varepsilon_1}{\varepsilon}\left[x^2 + \nu_2(1-x)^2 + \frac{2x\nu_2(1-x)}{\nu_2 x + 1 - x}\right],\ \nu_2 = \frac{\operatorname{tg}\delta_2\varepsilon_2}{\operatorname{tg}\delta_1\varepsilon_1}. \tag{13}$$

We have obtained the expressions (11) and (13) for the dielectric loss tangent of a binary system with a structure having interpenetrating components. The inputs for (3), (11), and (13) are the volume concentration m_2 and the quantities ρ_1, ρ_2, ε_1, ε_2, tan δ_1, and tan δ_2. The above method was used to calculate the dielectric loss tangent of porous polymer materials [5] and liquid binary solutions [6].

The results were compared to the experimental data [5, 6] and are shown in Tables 1 and 2. The relative error does not exceed 10% with a confidence of 0.95.

TABLE 1. Comparison of the Calculated and Tabulated Values of tan δ for Several Polymers

Material	Density, kg/m^3	$tg\delta_{calc.} \cdot 10^{-3}$	$tg\delta_{expt} \cdot 10^{-3}$	Rel. error, %
Polyurethane foam:				
PUF-305A	45	4,4	5	10
	150	7,2	8	—3
	250	8,5	8	—6
PUF-307	100	2,94	3	2
	200	6,1	6	—2
PUF-314	55	2,52	2,5	—1
	250	4,3	4,0	—8
Polytetrafluorethylene [4]	300	0,169	0,181	7

TABLE 2. Comparison of the Calculated and Tabulated Values of tan δ for Binary Solutions

t, °C	m_1	$tg\delta_{expt.}$	$tg\delta_{calc.}$	Rel. error, %
		Chloroform—acetone		
40	10	0,538	0,516	4,2
	30	0,598	0,530	11,4
	50	0,658	0,598	9,4
	70	0,501	0,487	3
	90	0,453	0,415	8,4
		Chloroform—2-buthanone		
40	10	0,994	0,956	4
	30	0,76	0,76	0
	50	0,633	0,632	0,2
	70	0,496	0,526	—6
	90	0,388	0,405	—4,4
		Chloroform-pyridine		
30	30	0,425	0,462	—8,7
	50	0,444	0,440	1
	70	0,429	0,402	6,3
		Acetone—hexane		
20	15	0,186	0,183	2
	30	0,353	0,310	12
	50	0,435	0,408	6,2
	70	0,506	0,474	6,3
		Chlorobenzene—xylene		
20	5	$9,7 \cdot 10^{-3}$	$8,8 \cdot 10^{-3}$	9
	25	$19,4 \cdot 10^{-3}$	$17,1 \cdot 10^{-3}$	12,4
	50	$32,9 \cdot 10^{-3}$	$27,3 \cdot 10^{-3}$	17

NOTATION

Λ, transport coefficient; ε, dielectric constant; tan δ, loss tangent; σ, electric conductivity; ρ, resistivity of the material; ω, frequency; R, resistance; C, capacitance; S, cross-sectional area; L, length of the elementary cell; Δ, geometrical parameter of the elementary cell; m_1, m_2, volume concentrations of the first and second components.

LITERATURE CITED

1. G. N. Dul'nev, Transport Coefficients in Nonuniform Media (Textbook) [in Russian], Leningrad (1979).
2. J. L. Altman, Microwave Circuits, Van Nostrand, New York (1964).
3. A. A. Brant, Study of Dielectrics at Ultra-High Frequencies [in Russian], Moscow (1963).
4. A. R. von Hippel (ed.), Dielectric Materials and Applications, MIT Technology Press, Cambridge, Mass., Wiley, New York (1954).
5. V. V. Barsova and N. G. Shilet, Tr. Vsesoyuz. Inst. Ogneuporov, 24-28 (1975).
6. Ya. Yu. Akhadov, Dielectric Properties of Binary Solutions [in Russian], Moscow (1977).